

CSC242: Intro to AI

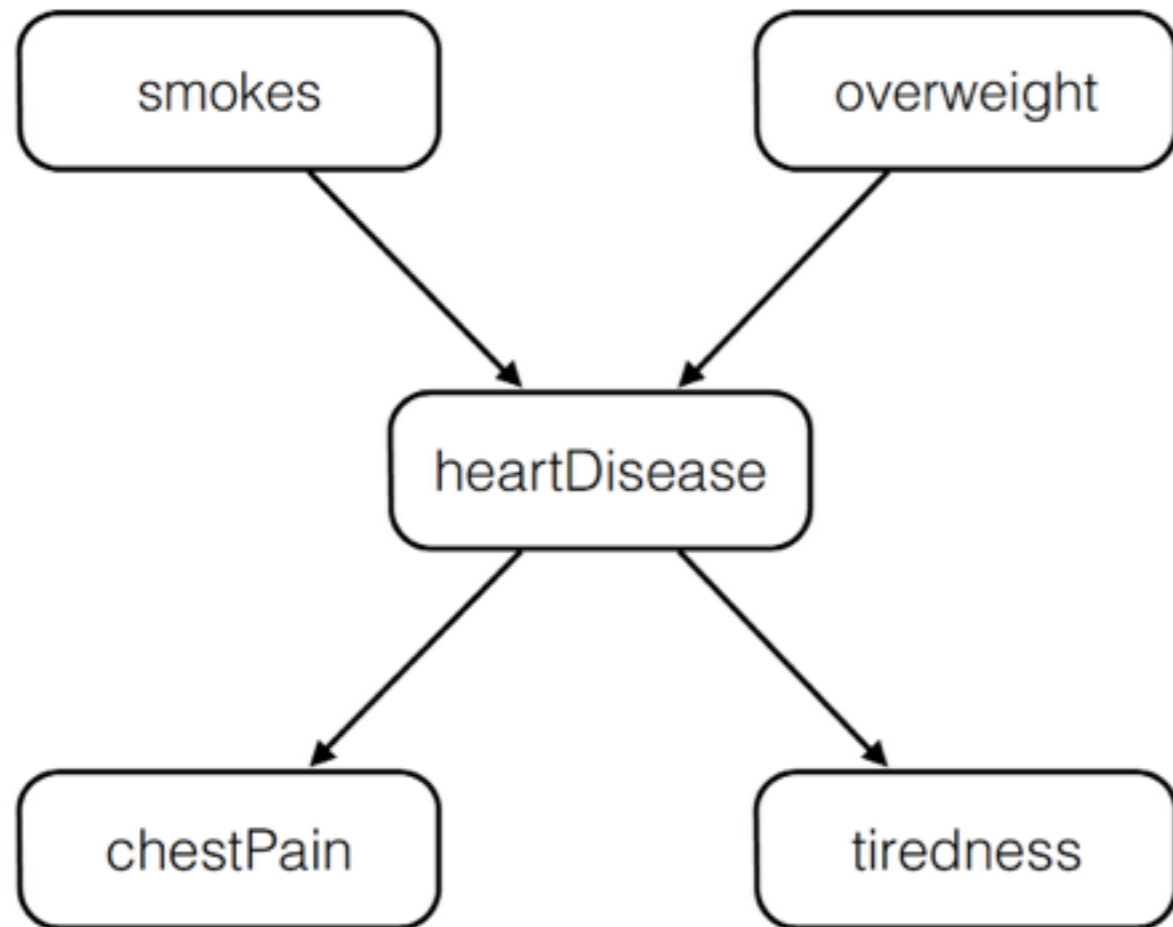
Lecture 21

Wrap Up

Othello Championship

Review Problems

#9: Bayesian Networks



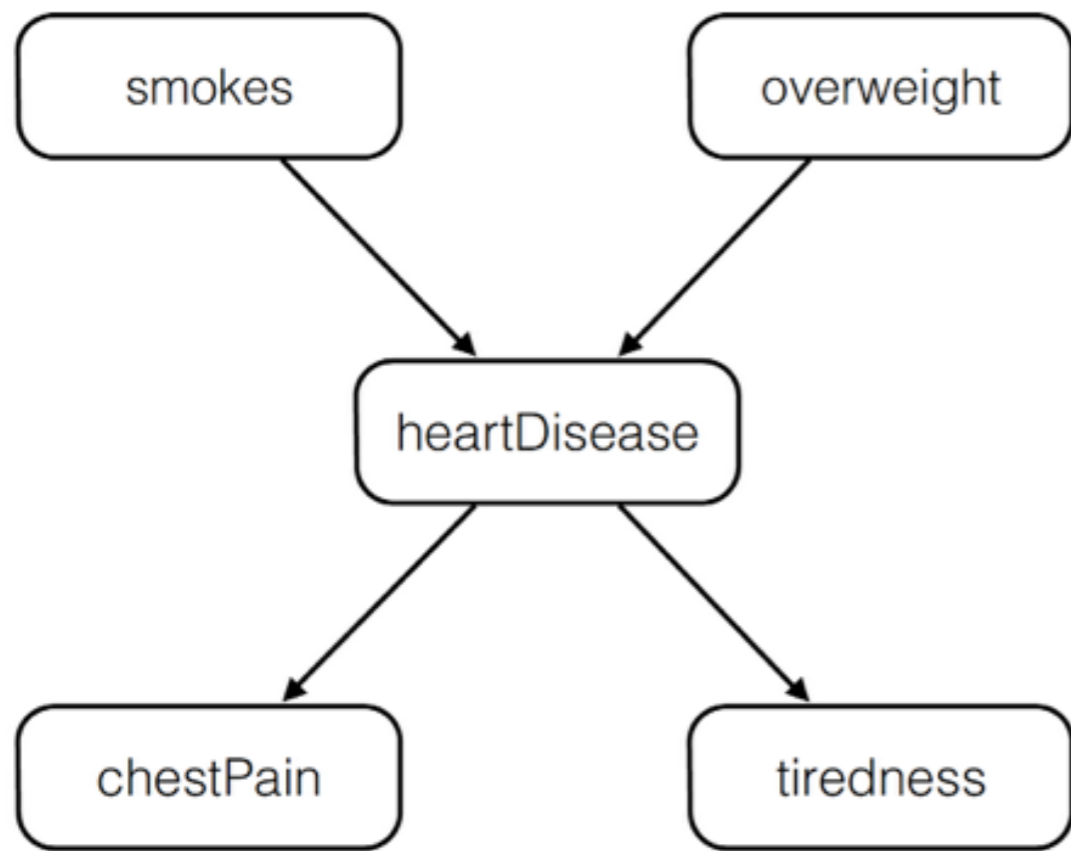
$$P(\text{smokes}) = 0.2$$

$$P(\text{overweight}) = 0.6$$

$$P(\text{heartDisease}) = \begin{cases} & \text{smokes} & \neg\text{smokes} \\ \text{overweight} & 0.5 & 0.3 \\ \neg\text{overweight} & 0.2 & 0.1 \end{cases}$$

	heartDisease	\neg heartDisease
P(chestPain)	0.3	0.1

	heartDisease	\neg heartDisease
P(tiredness)	0.5	0.4



$$P(\text{smokes}) = 0.2$$

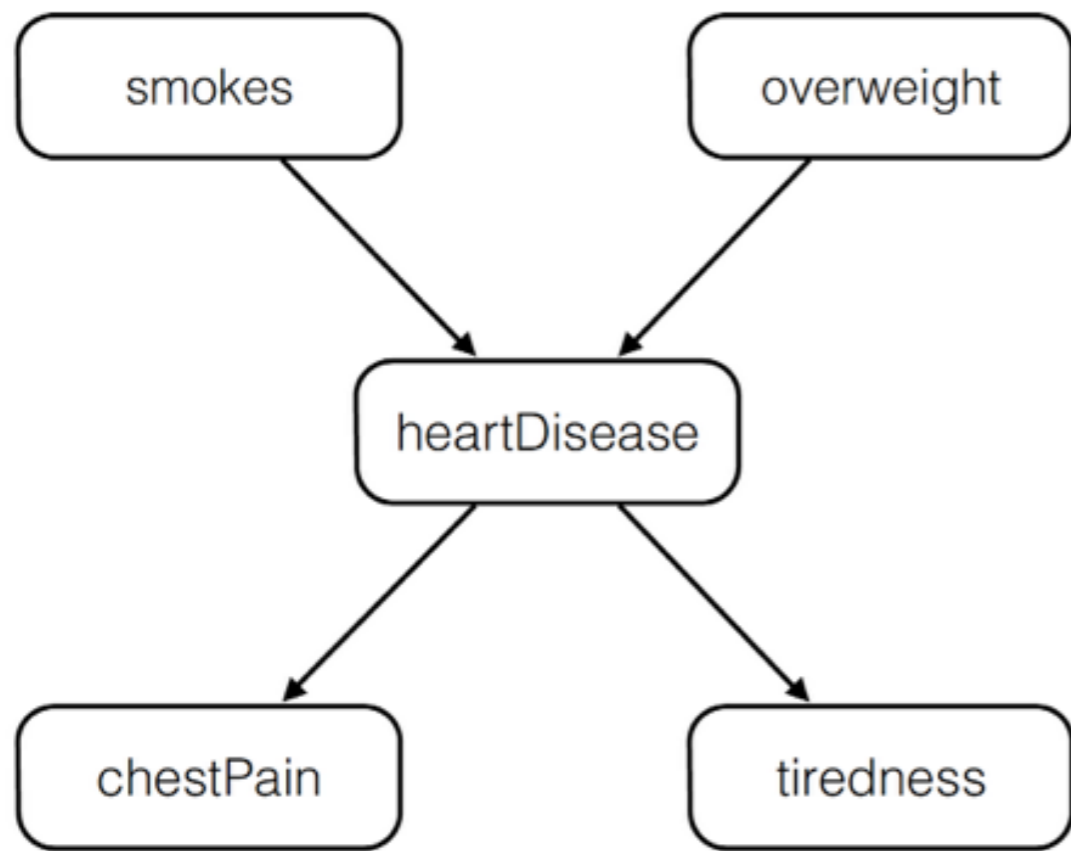
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	heartDisease	\neg heartDisease
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Calculate $P(\text{heartDisease})$



$$P(\text{smokes}) = 0.2$$

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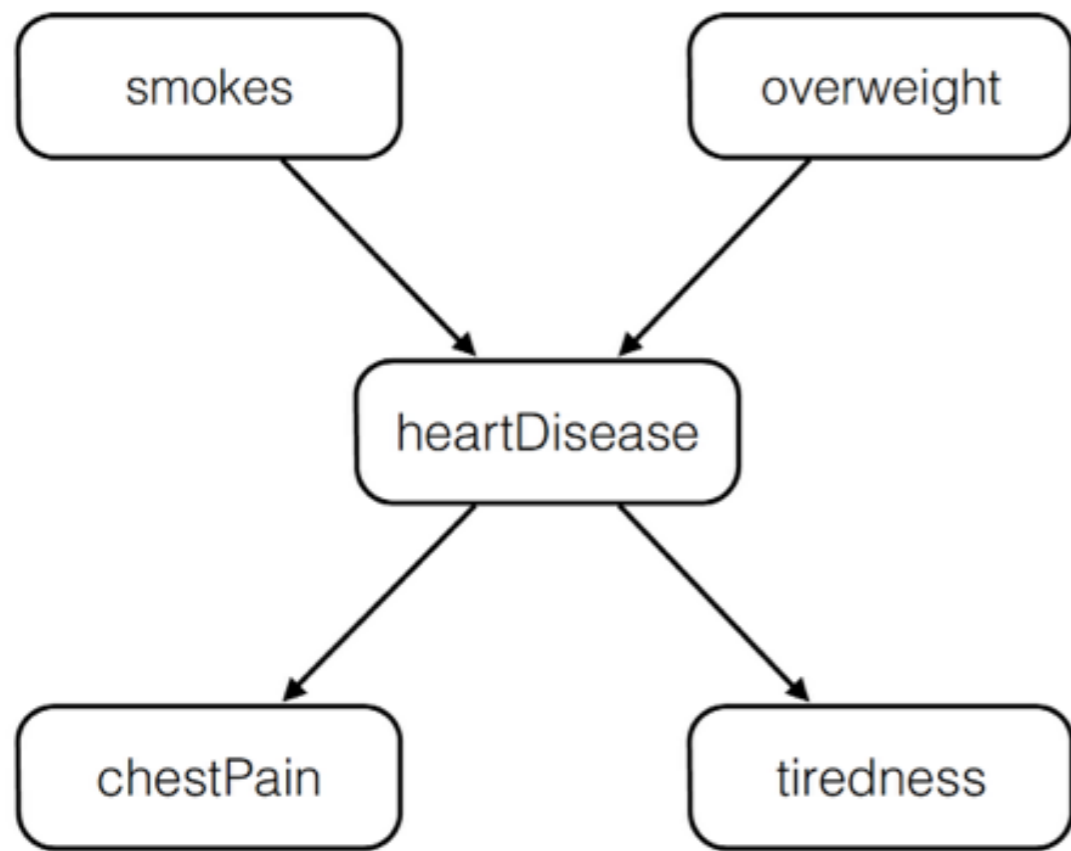
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Calculate $P(\text{heartDisease})$

Solution: Condition on cases:



$$P(\text{smokes}) = 0.2$$

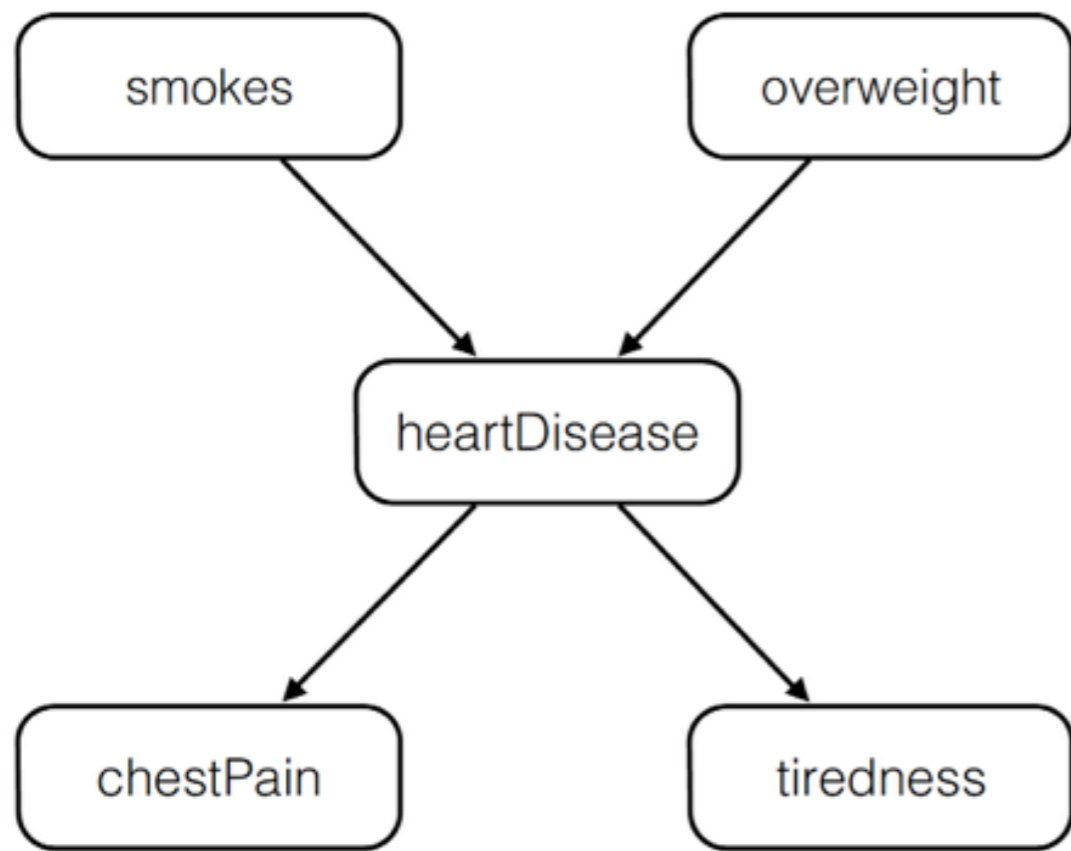
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$$\begin{aligned}
 P(h) &= P(h|s, o)P(s, o) + P(h|\neg s, o)P(\neg s, o) \\
 &\quad + P(h|s, \neg o)P(s, \neg o) + P(h|\neg s, \neg o)P(\neg s, \neg o)
 \end{aligned}$$



$$P(\text{smokes}) = 0.2$$

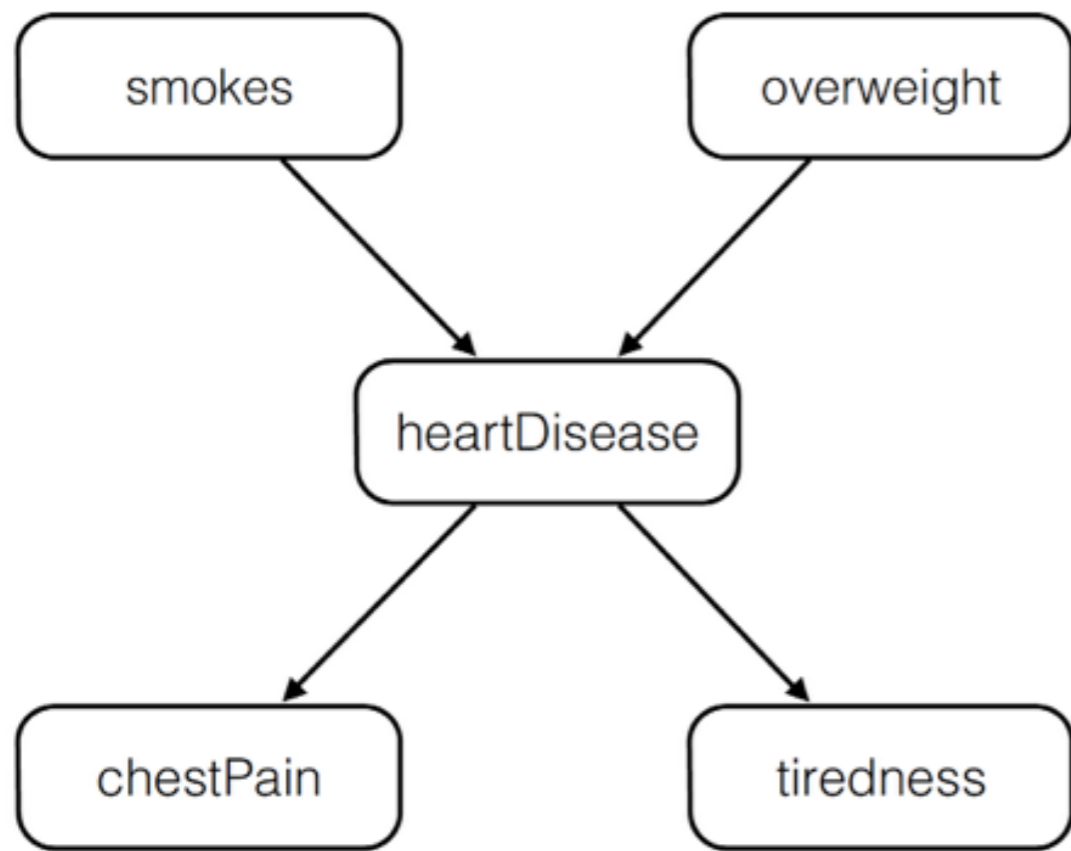
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 &= P(h|s, o)P(s)P(o) + P(h|\neg s, o)P(\neg s)P(o) \\
 &\quad + P(h|s, \neg o)P(s)P(\neg o) + P(h|\neg s, \neg o)P(\neg s)P(\neg o)
 \end{aligned}$$



$$P(\text{smokes}) = 0.2$$

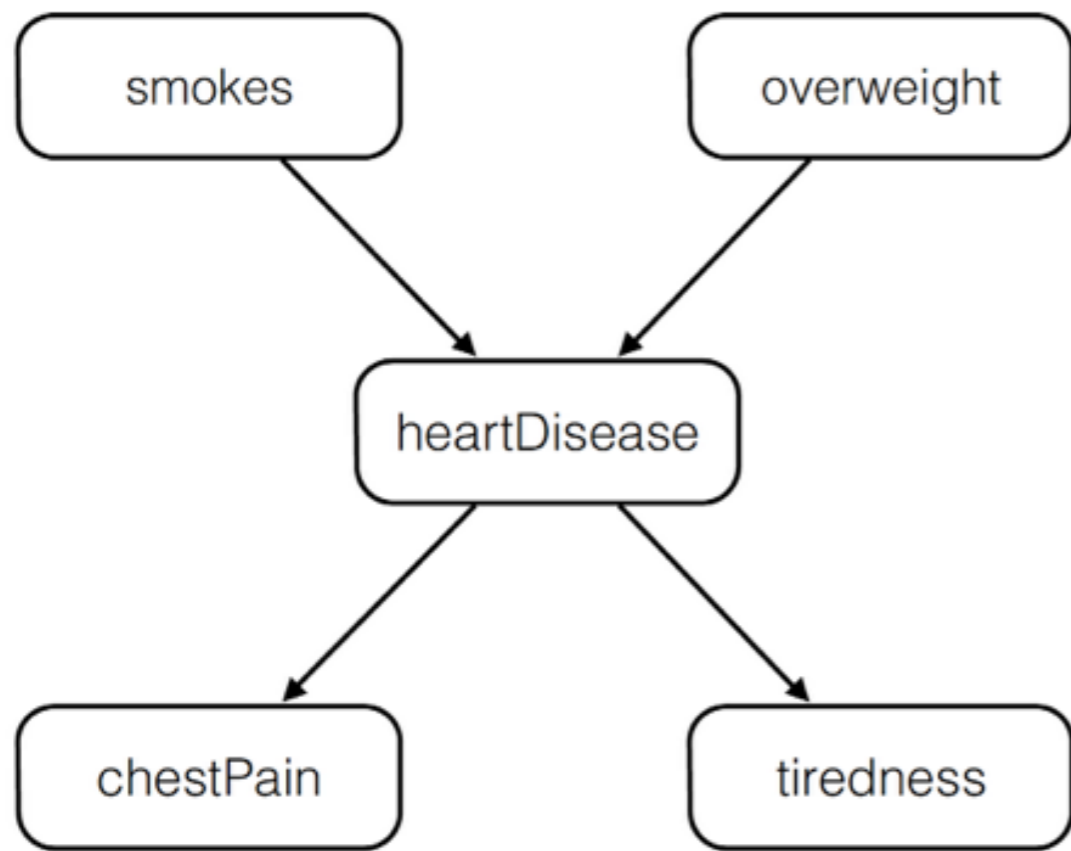
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 &= P(h|s, o)P(s)P(o) + P(h|\neg s, o)P(\neg s)P(o) \\
 &\quad + P(h|s, \neg o)P(s)P(\neg o) + P(h|\neg s, \neg o)P(\neg s)P(\neg o) \\
 &= (0.5)(0.2)(0.6) + (0.3)(1 - 0.2)(0.6) \\
 &\quad + (0.2)(0.2)(1 - 0.6) + (0.1)(1 - 0.2)(1 - 0.6) \\
 &= 0.252
 \end{aligned}$$



$$P(\text{smokes}) = 0.2$$

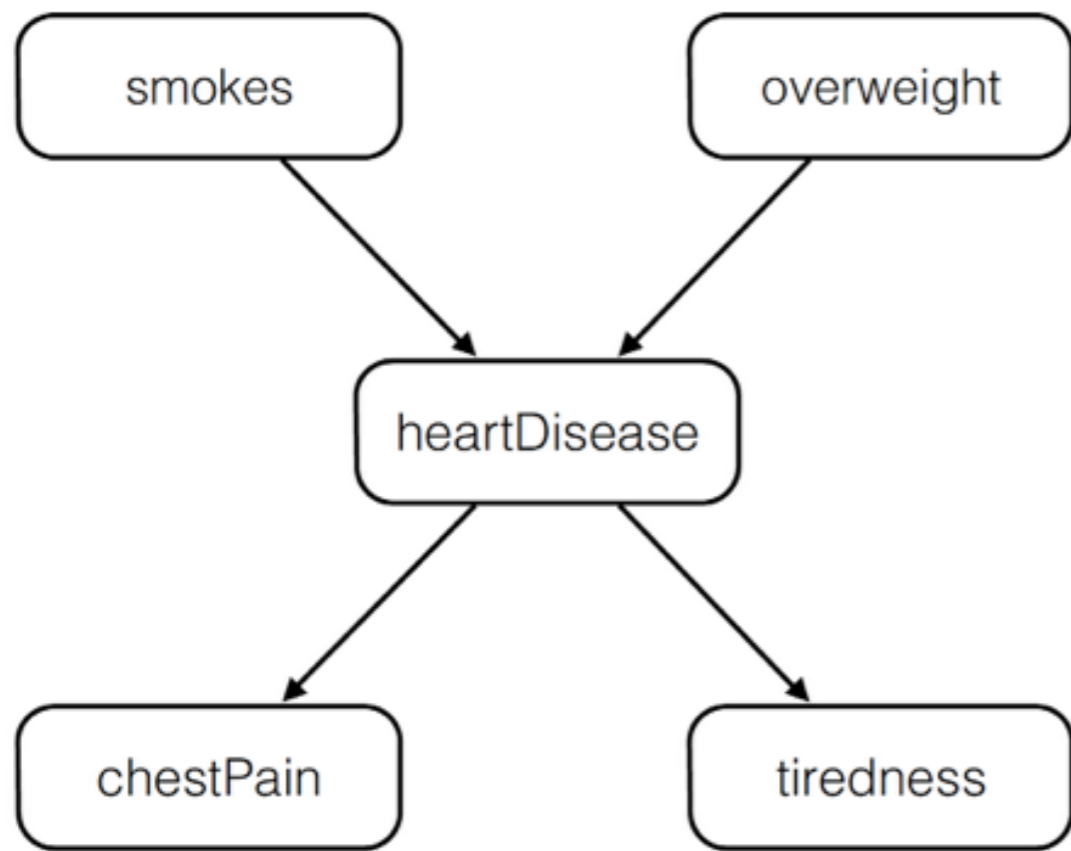
$$P(\text{overweight}) = 0.6$$

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	heartDisease	\neg heartDisease
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	heartDisease	\neg heartDisease
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Calculate $P(\text{chestPain})$.



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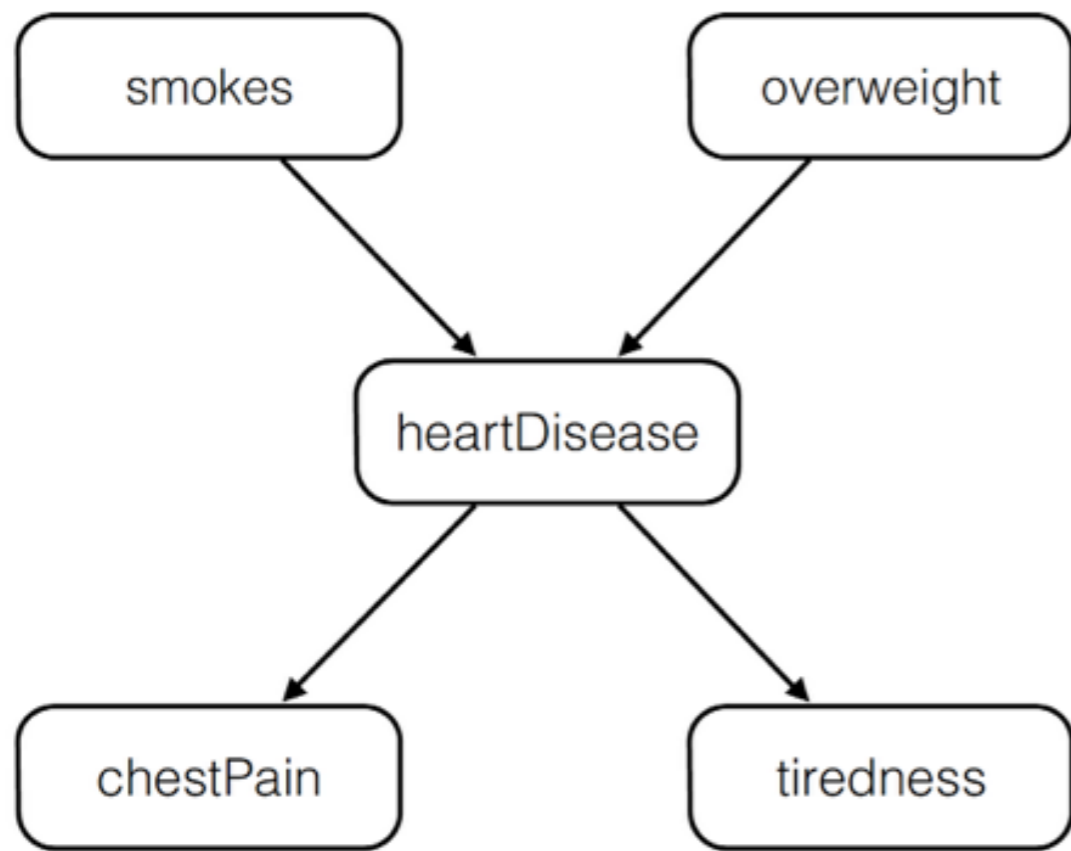
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P(tiredness)	0.5	0.4

Calculate $P(\text{chestPain})$.

Solution: Condition on cases:



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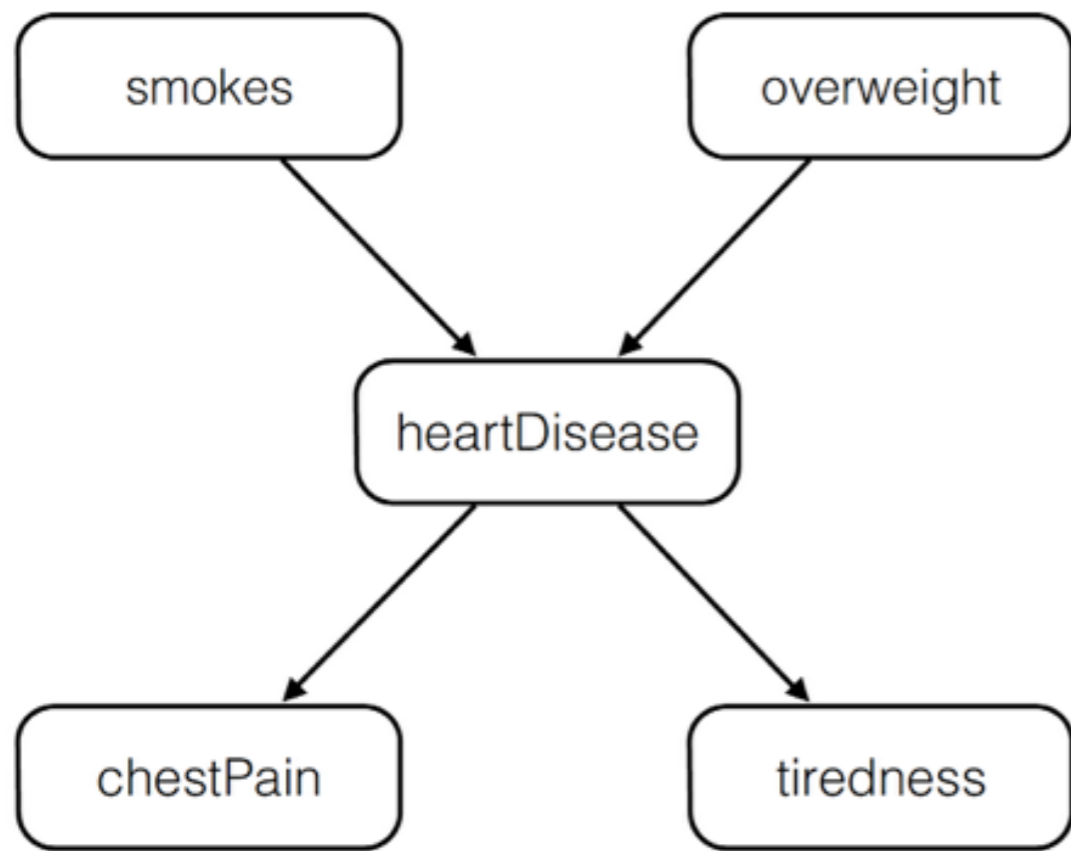
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	heartDisease	\neg heartDisease
$P(\text{chestPain})$	0.3	0.1

	heartDisease	\neg heartDisease
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$$P(c) = P(c|h)P(h) + P(c|\neg h)P(\neg h)$$



$$P(\text{smokes}) = 0.2$$

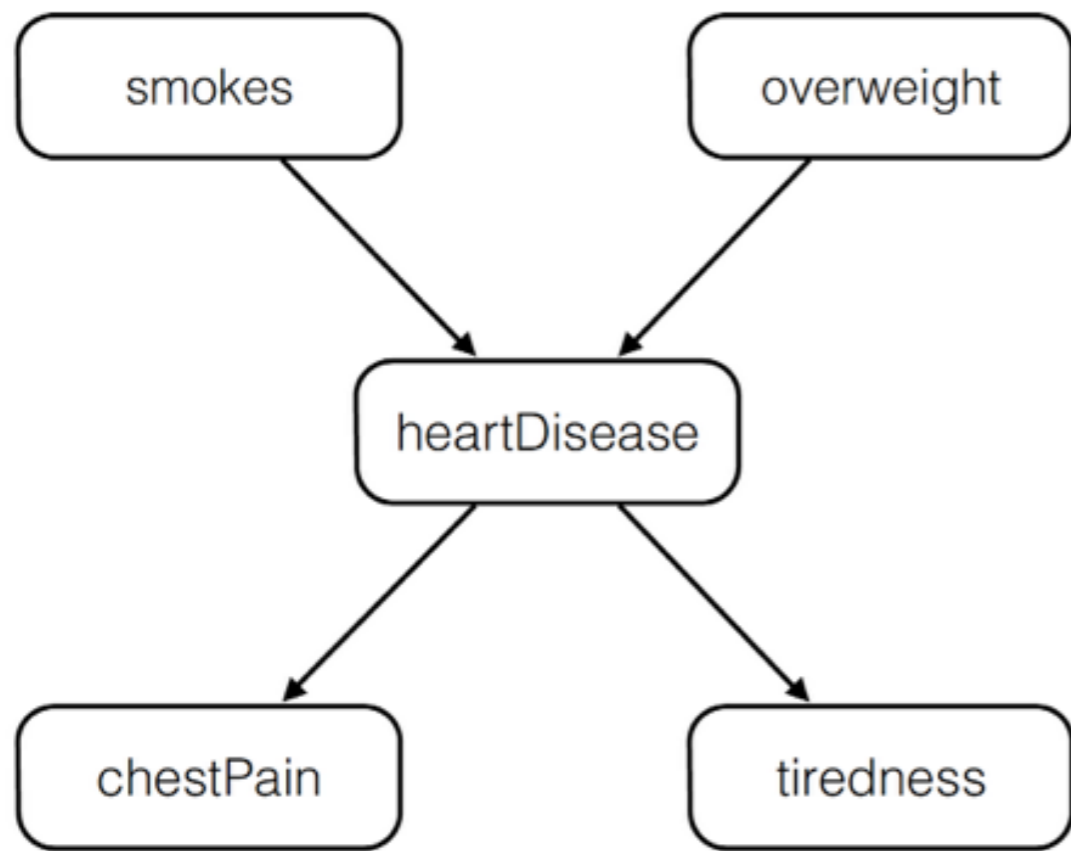
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	heartDisease	\neg heartDisease
$P(\text{chestPain})$	0.3	0.1

	heartDisease	\neg heartDisease
$P(\text{tiredness})$	0.5	0.4

$$\begin{aligned}
 P(c) &= P(c|h)P(h) + P(c|\neg h)P(\neg h) \\
 &= (0.3)(0.252) + (0.1)(1 - 0.252)
 \end{aligned}$$



$$P(\text{smokes}) = 0.2$$

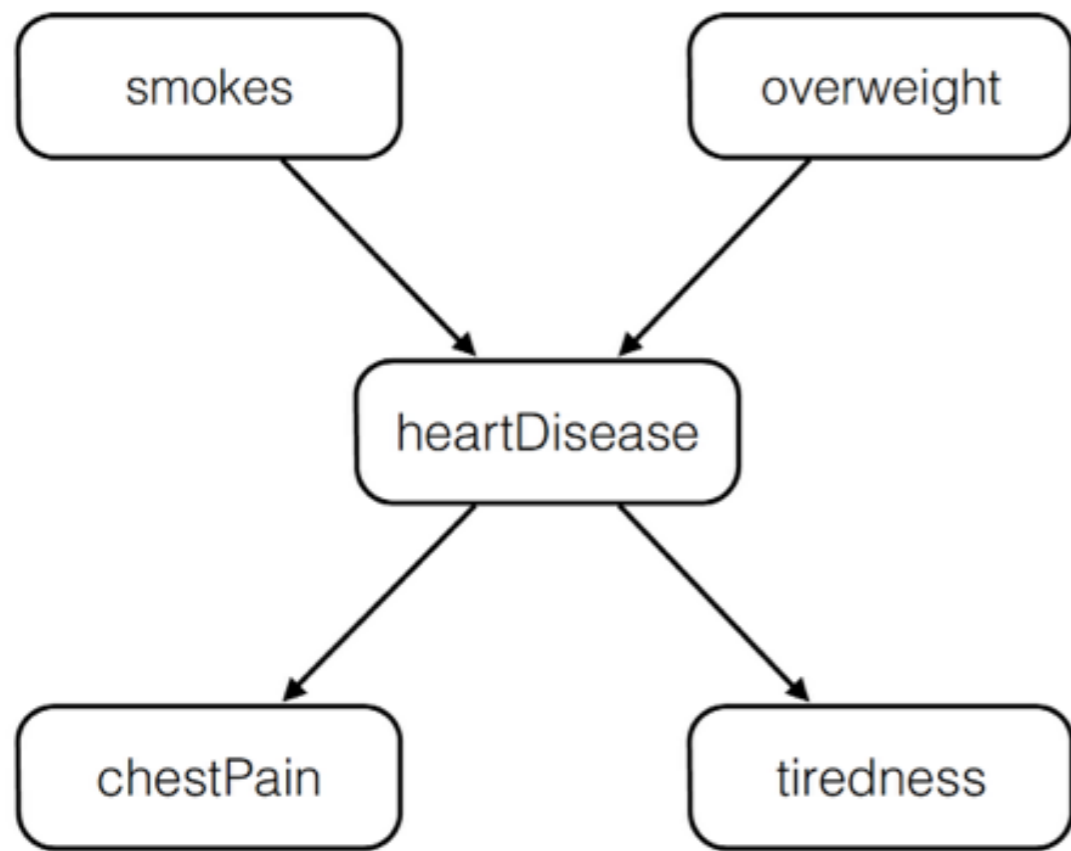
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$P(\text{chestPain})$	0.3	0.1

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$$\begin{aligned}
 P(c) &= P(c|h)P(h) + P(c|\neg h)P(\neg h) \\
 &= (0.3)(0.252) + (0.1)(1 - 0.252) \\
 &= 0.1504
 \end{aligned}$$



$$P(\text{smokes}) = 0.2$$

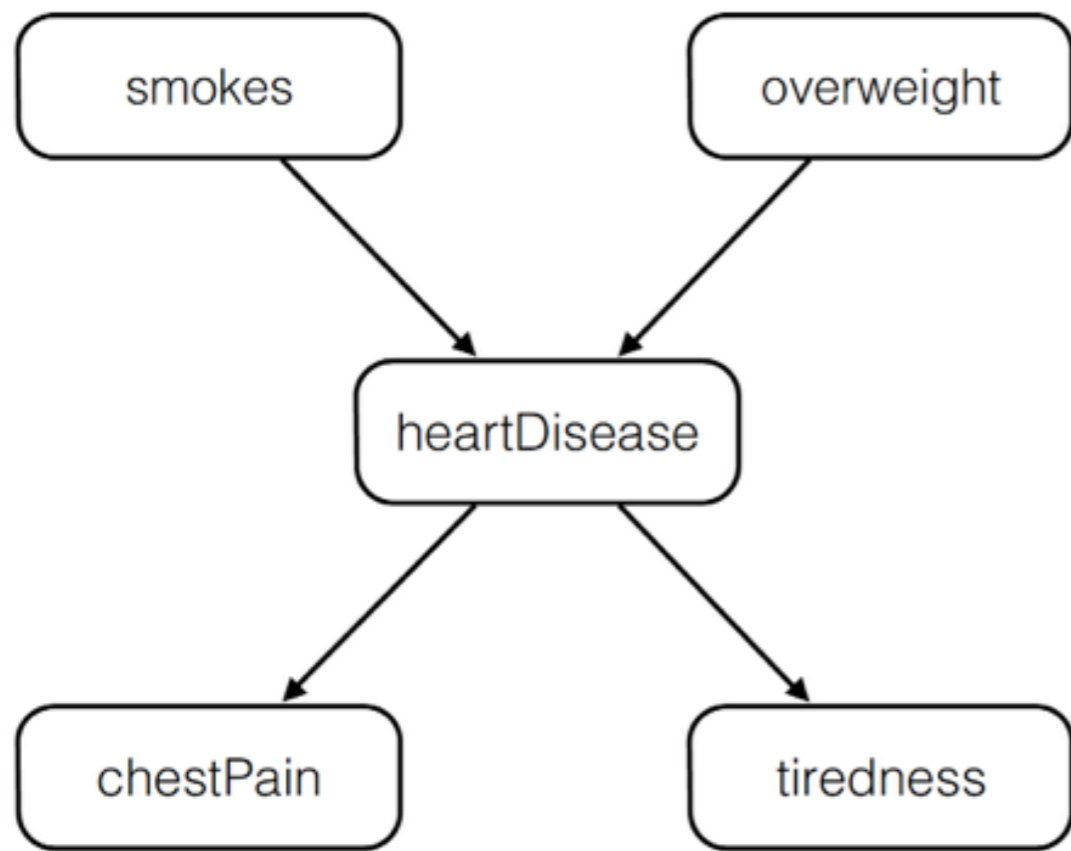
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Calculate $P(\text{heartDisease} \mid \text{chestPain})$.



$$P(\text{smokes}) = 0.2$$

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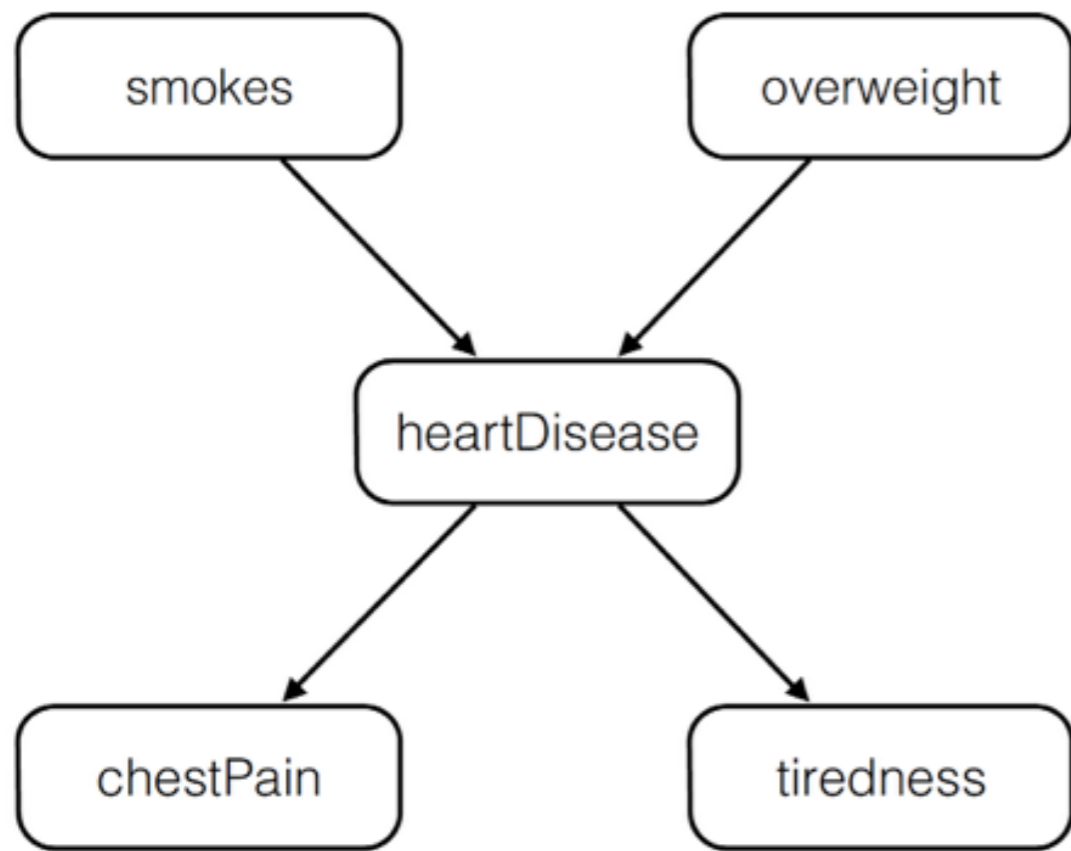
$$P(\text{heartDisease}) = \begin{cases} & \begin{array}{c|c} & \text{smokes} & \neg\text{smokes} \\ \hline \text{overweight} & 0.5 & 0.3 \\ \hline \neg\text{overweight} & 0.2 & 0.1 \end{array} \end{cases}$$

	heartDisease	\neg heartDisease
P(chestPain)	0.3	0.1

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Calculate $P(\text{heartDisease} \mid \text{chestPain})$.

Solution: Use Bayes law:



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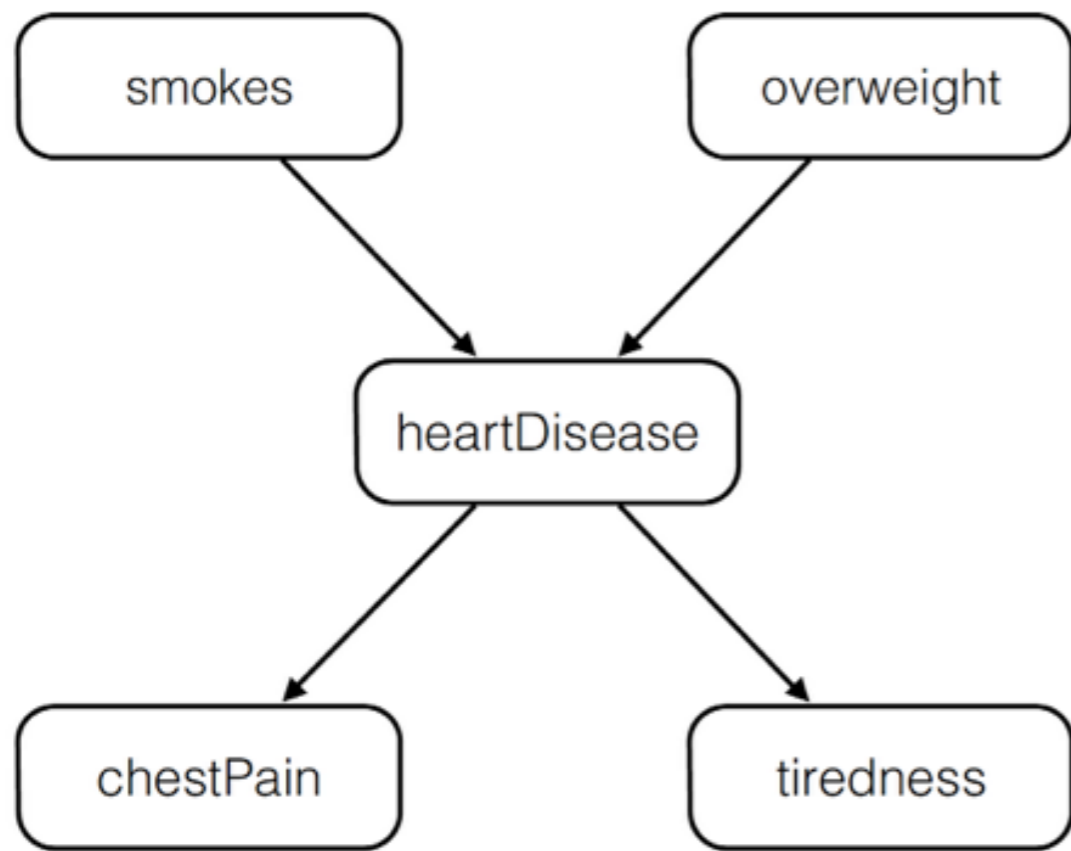
$$P(\text{overweight}) = 0.6$$

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	heartDisease	\neg heartDisease
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	heartDisease	\neg heartDisease
P(tiredness)	0.5	0.4

$$P(h|c) = P(c|h)P(h)/P(c)$$



$$P(\text{smokes}) = 0.2$$

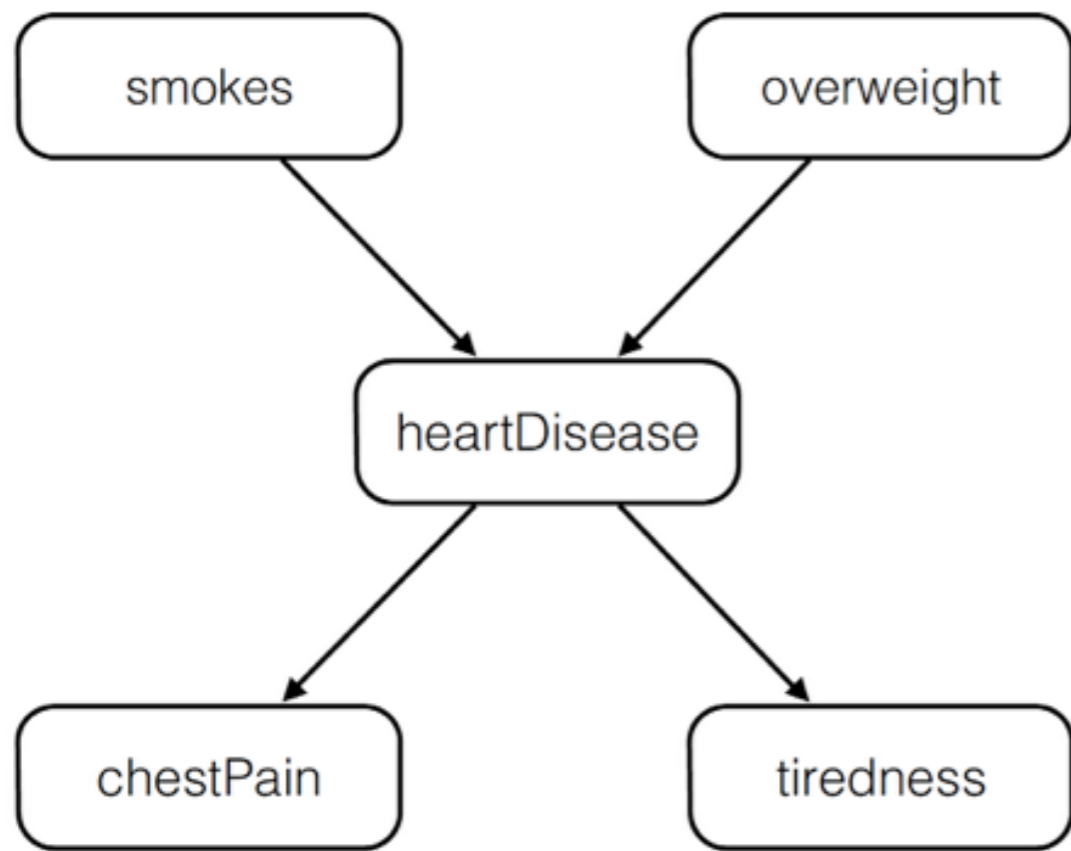
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P(chestPain)	0.3	0.1

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$$\begin{aligned}
 P(h|c) &= P(c|h)P(h)/P(c) \\
 &= (0.3)(0.252)/(0.1504)
 \end{aligned}$$



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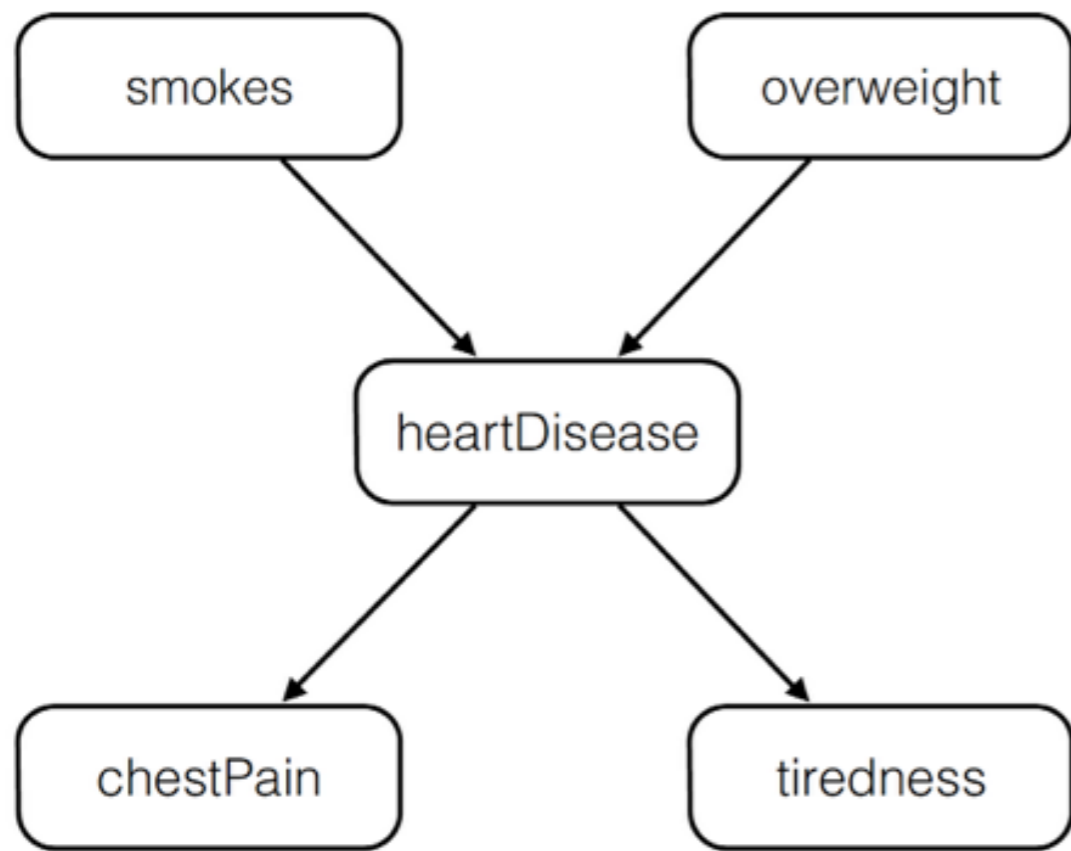
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$$\begin{aligned}
 P(h|c) &= P(c|h)P(h)/P(c) \\
 &= (0.3)(0.252)/(0.1504) \\
 &= 0.5027
 \end{aligned}$$



$$P(\text{smokes}) = 0.2$$

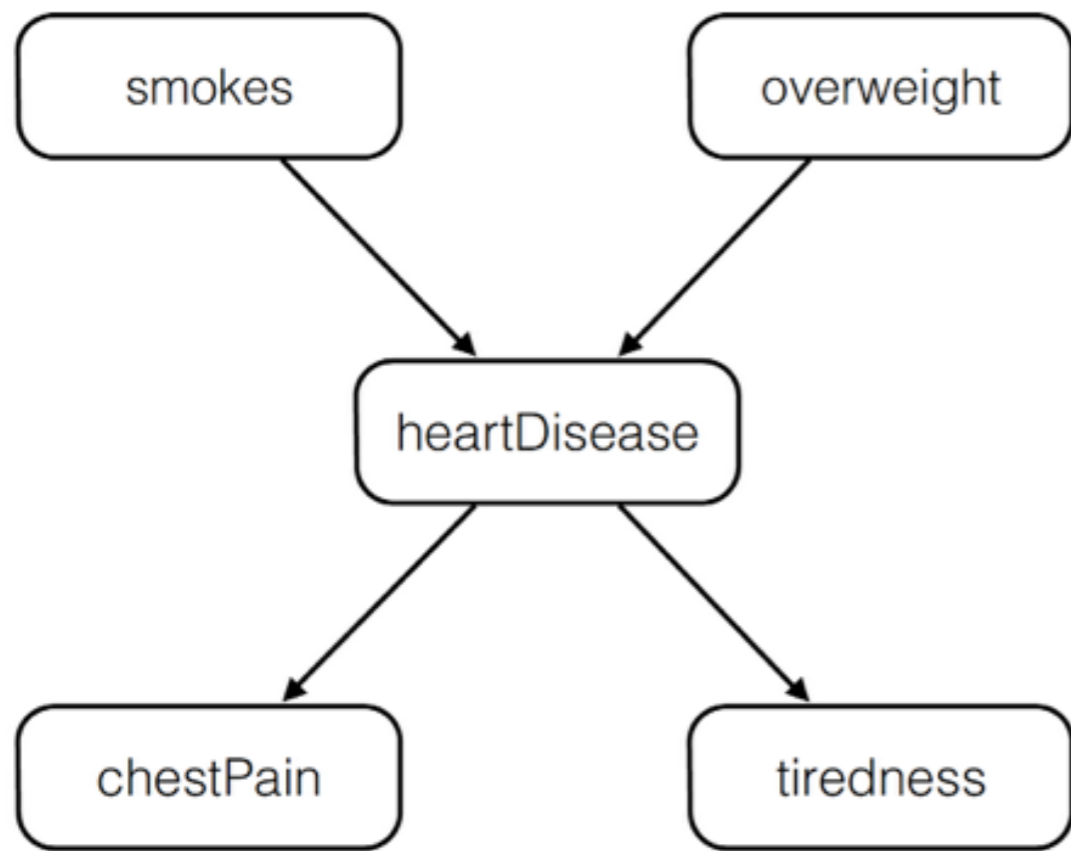
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	heartDisease	\neg heartDisease
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Calculate $P(\text{heartDisease} \mid \text{chestPain}, \neg\text{overweight})$



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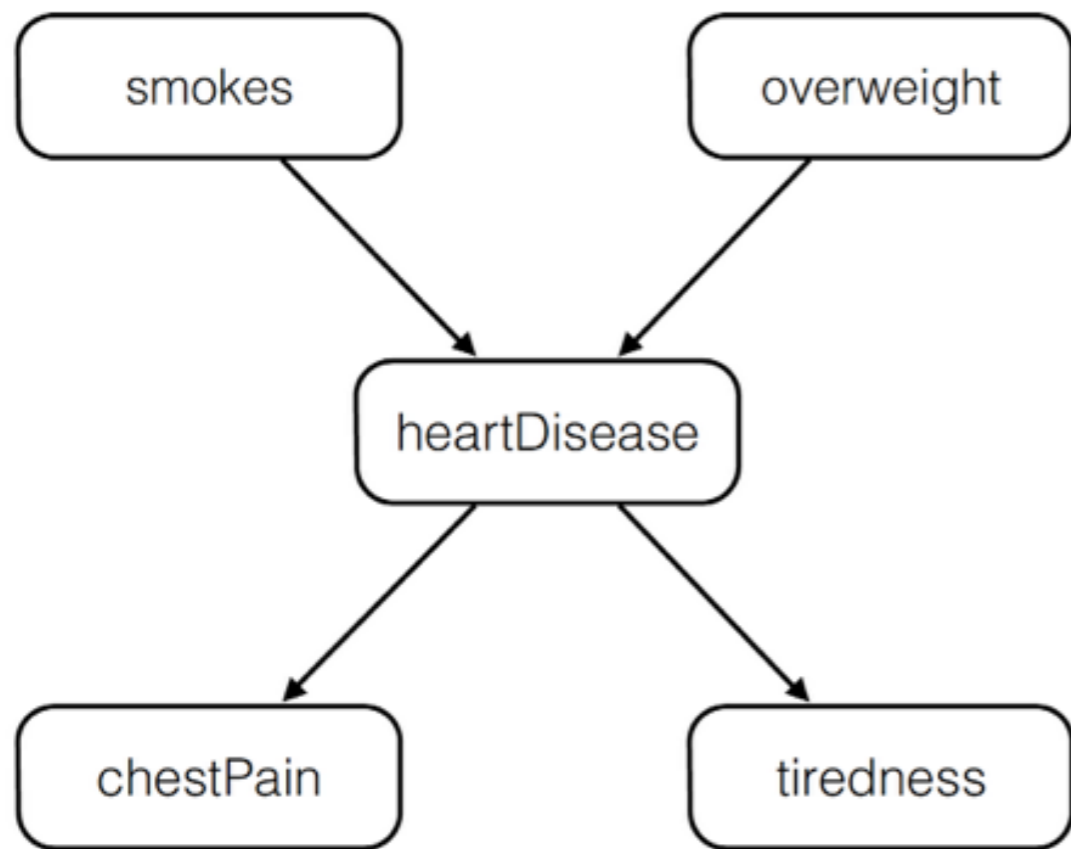
	heartDisease	\neg heartDisease
$P(\text{chestPain})$	0.3	0.1

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$P(\text{tiredness})$	0.5	0.4

Calculate $P(\text{heartDisease} \mid \text{chestPain}, \neg\text{overweight})$

Solution: By Bayes law:

$$P(h|c, \neg o) = P(c|h, \neg o)P(h|\neg o)/P(c|\neg o)$$



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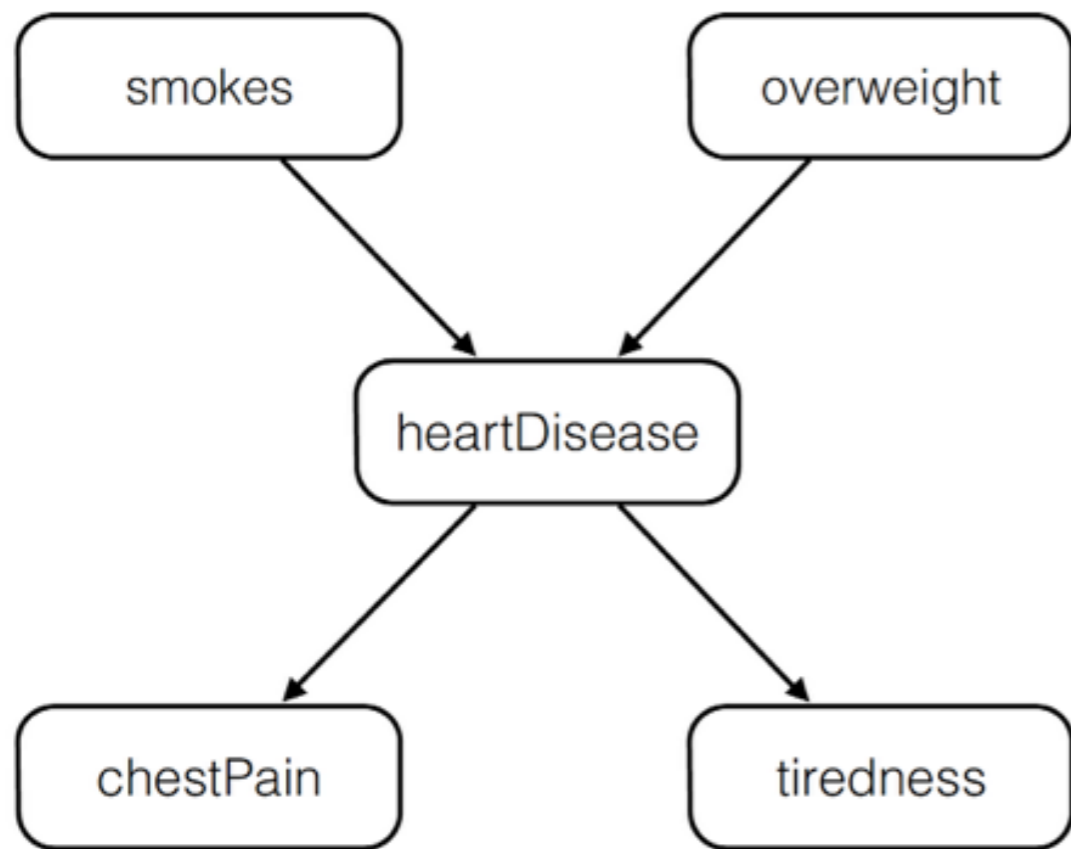
Calculate $P(\text{heartDisease} \mid \text{chestPain}, \neg\text{overweight})$

Solution: By Bayes law:

$$P(h|c, \neg o) = P(c|h, \neg o)P(h|\neg o)/P(c|\neg o)$$

Because chestPain is independent of overweight given heartDisease:

$$P(h|c, \neg o) = P(c|h)P(h|\neg o)/P(c|\neg o)$$



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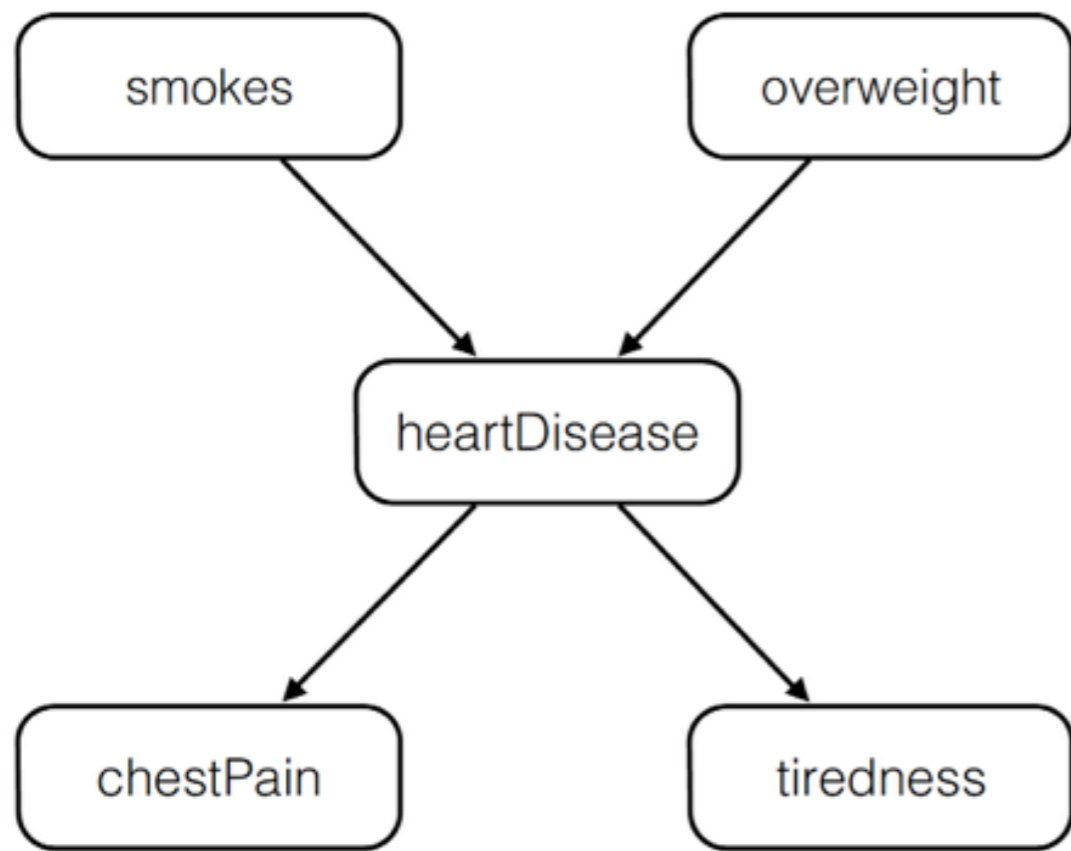
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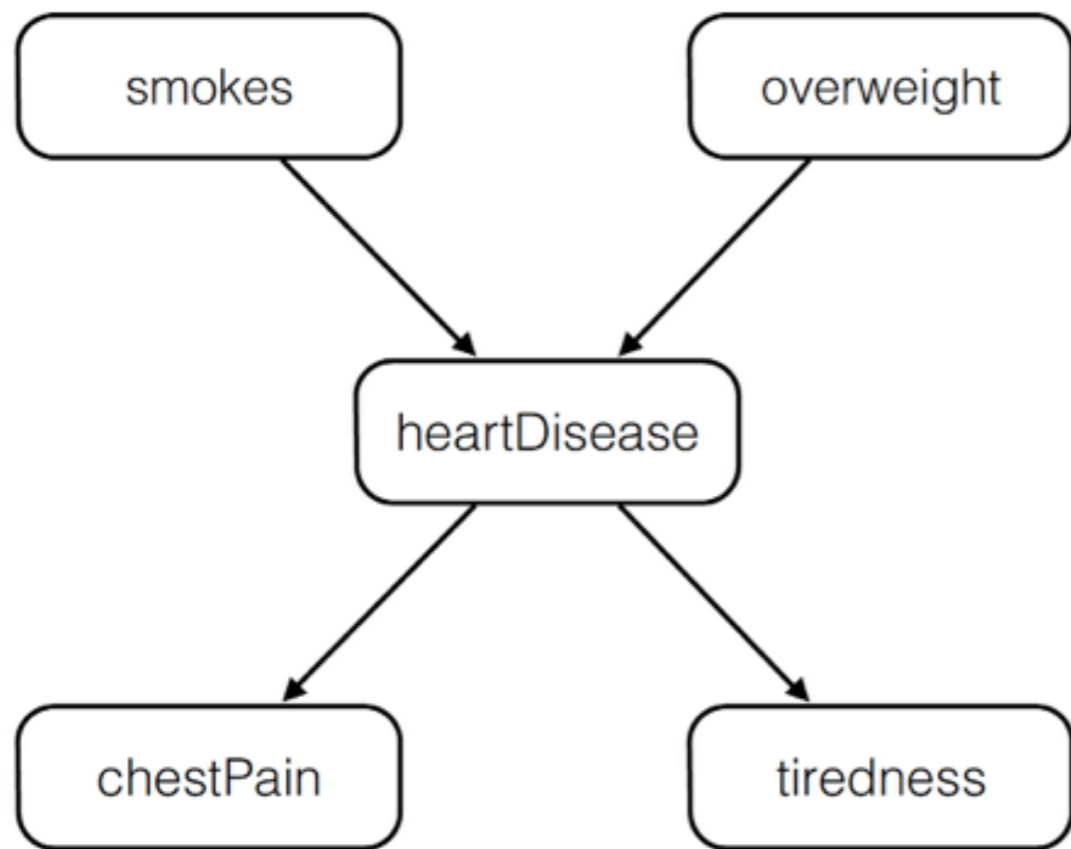
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	heartDisease	\neg heartDisease
$P(\text{chestPain})$	0.3	0.1

	heartDisease	\neg heartDisease
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Calculate $P(\text{heartDisease} \mid \text{chestPain}, \neg\text{overweight})$

$$\begin{aligned}
 P(h \mid \neg o) &= P(h \mid \neg o, s)P(s) + P(h \mid \neg o, \neg s)P(\neg s) \\
 &= (0.2)(0.2) + (0.1)(1 - 0.2) \\
 &= 0.12
 \end{aligned}$$



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$$P(\text{heartDisease}) = \begin{cases} & \text{smokes} & \neg\text{smokes} \\ \text{overweight} & 0.5 & 0.3 \\ \neg\text{overweight} & 0.2 & 0.1 \end{cases}$$

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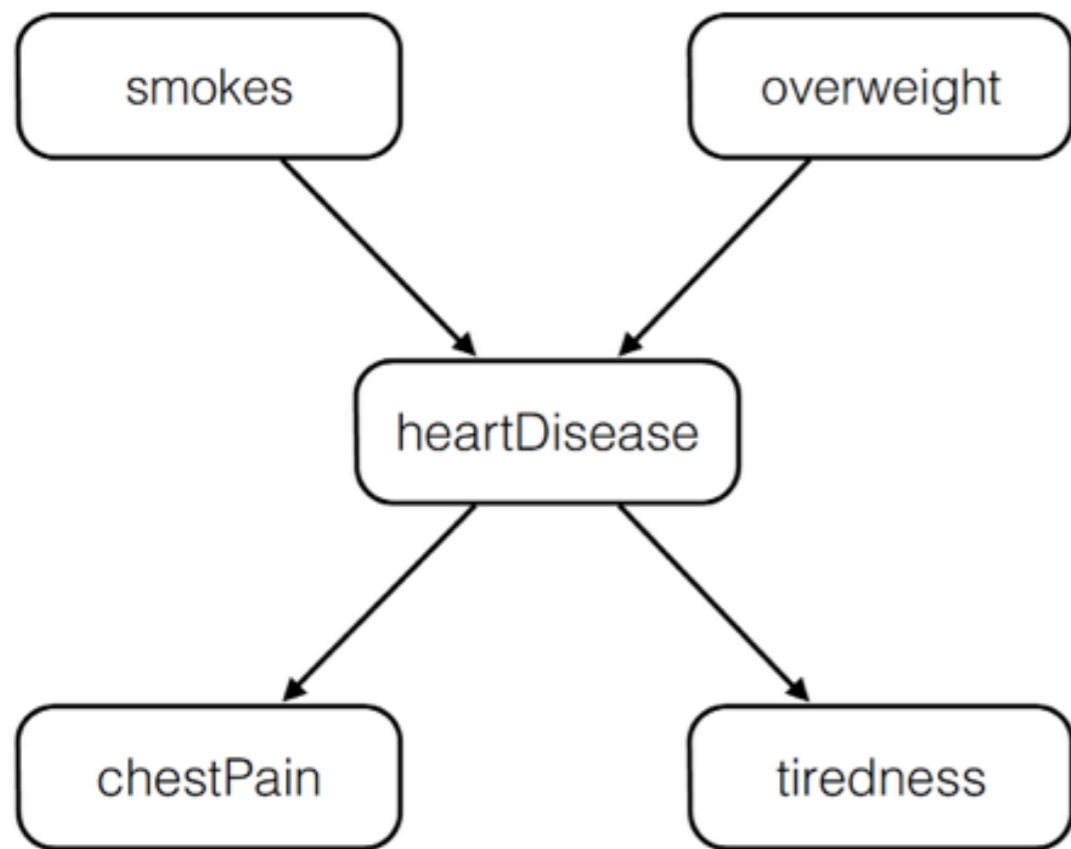
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Solution: By Bayes law:

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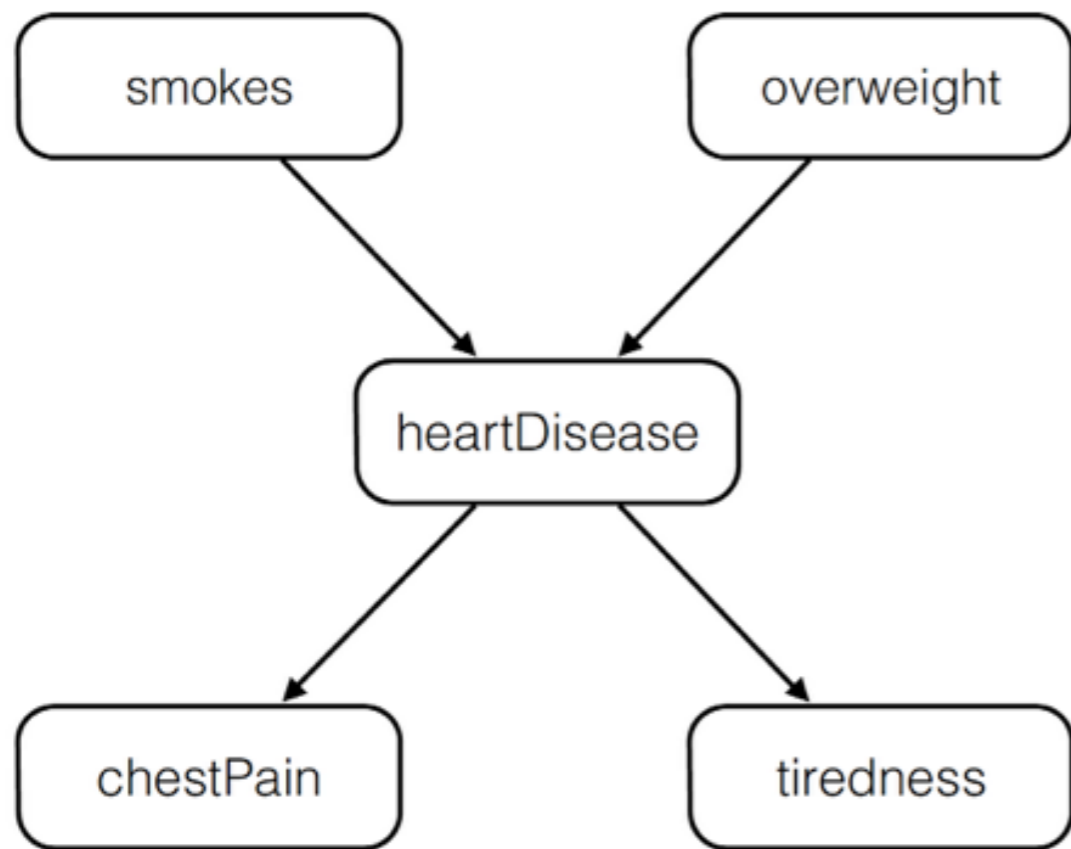
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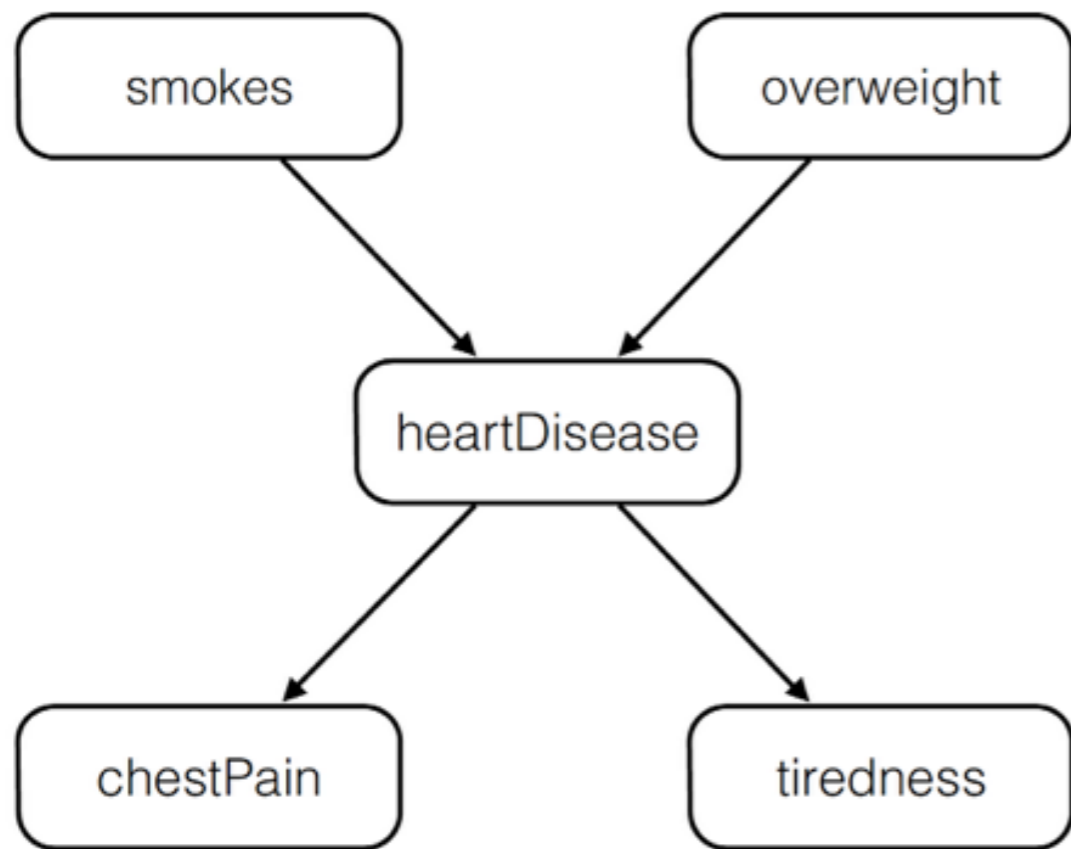
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	heartDisease	\neg heartDisease
$P(\text{chestPain})$	0.3	0.1

	heartDisease	\neg heartDisease
$P(\text{tiredness})$	0.5	0.4

Calculate $P(\text{heartDisease} \mid \text{chestPain}, \neg\text{overweight})$

We also need $P(c|\neg o)$. We can calculate this by conditioning on heartDisease, and then using the independence of chestPain from overweight given heartDisease.



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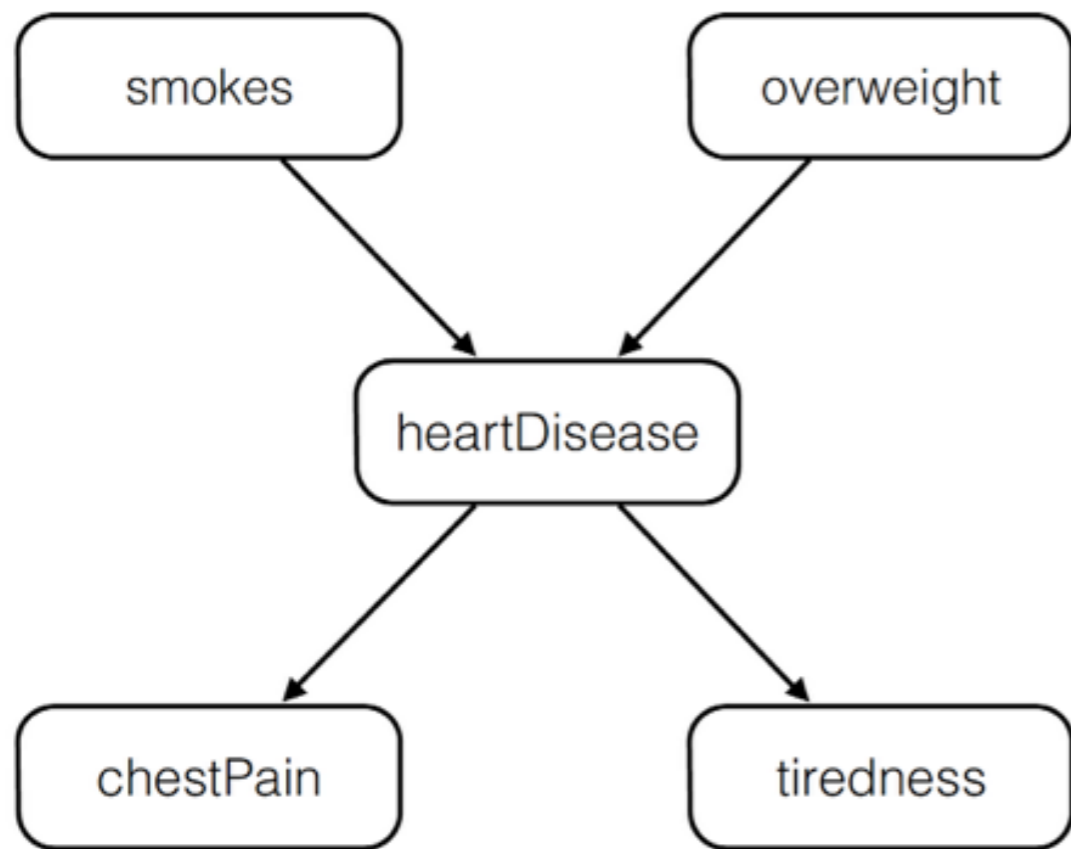
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We also need $P(c|\neg o)$. We can calculate this by conditioning on heartDisease, and then using the independence of chestPain from overweight given heartDisease.

$$P(c|\neg o) = P(c|\neg o, h)P(h) + P(c|\neg o, \neg h)P(\neg h)$$



$$P(\text{smokes}) = 0.2$$

$$P(\text{overweight}) = 0.6$$

$$P(\text{heartDisease}) = \begin{cases} & \text{smokes} & \neg\text{smokes} \\ \text{overweight} & 0.5 & 0.3 \\ \neg\text{overweight} & 0.2 & 0.1 \end{cases}$$

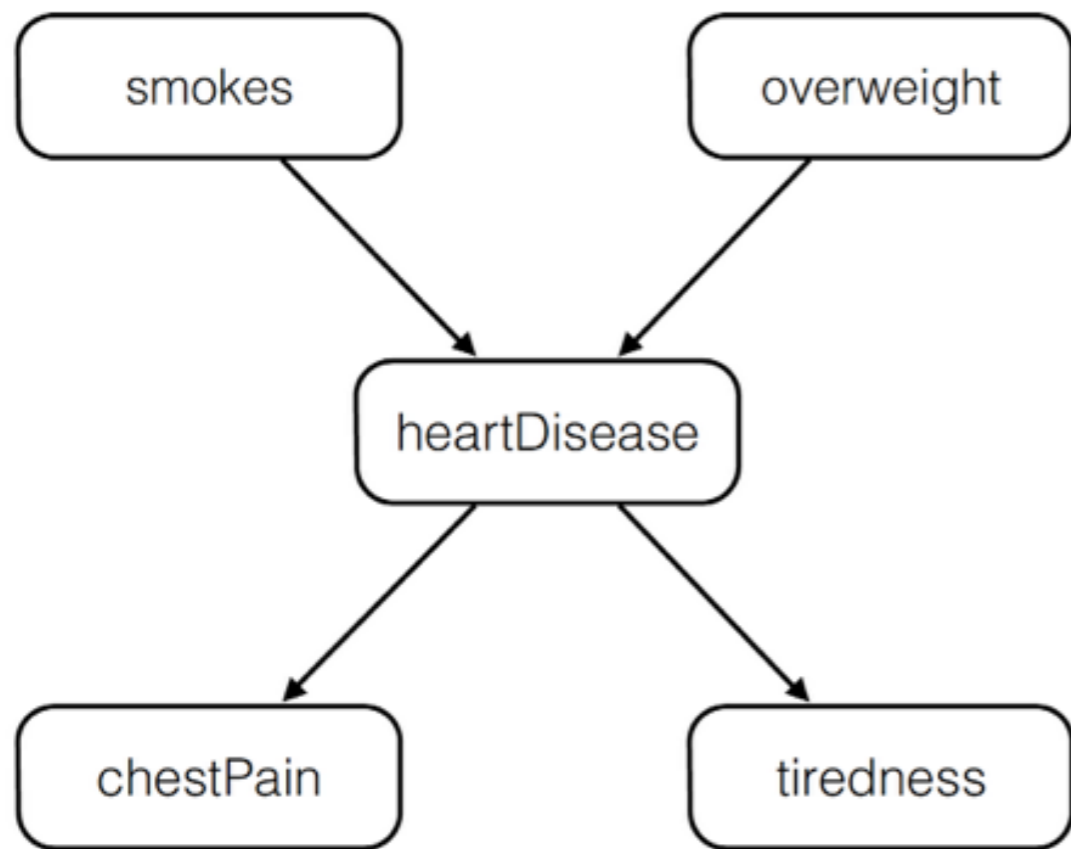
	heartDisease	\neg heartDisease
$P(\text{chestPain})$	0.3	0.1

	heartDisease	\neg heartDisease
$P(\text{tiredness})$	0.5	0.4

Calculate $P(\text{heartDisease} \mid \text{chestPain}, \neg\text{overweight})$

We also need $P(c|\neg o)$. We can calculate this by conditioning on heartDisease, and then using the independence of chestPain from overweight given heartDisease.

$$\begin{aligned} P(c|\neg o) &= P(c|\neg o, h)P(h) + P(c|\neg o, \neg h)P(\neg h) \\ &= P(c|h)P(h) + P(c|\neg h)P(\neg h) \end{aligned}$$



$$P(\text{smokes}) = 0.2$$

$$P(\text{overweight}) = 0.6$$

$$P(\text{heartDisease}) = \begin{cases} & \text{smokes} & \neg\text{smokes} \\ \text{overweight} & 0.5 & 0.3 \\ \neg\text{overweight} & 0.2 & 0.1 \end{cases}$$

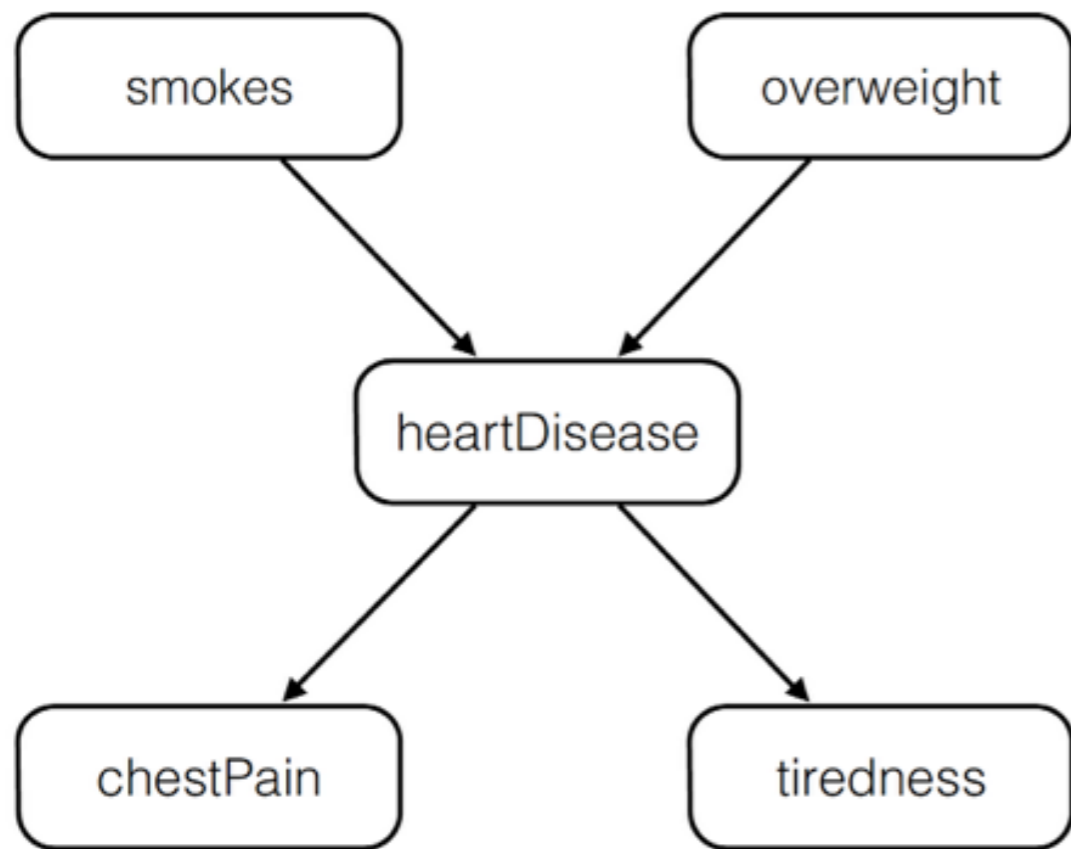
	heartDisease	\neg heartDisease
P(chestPain)	0.3	0.1

	heartDisease	\neg heartDisease
P(tiredness)	0.5	0.4

Calculate $P(\text{heartDisease} \mid \text{chestPain}, \neg\text{overweight})$

We also need $P(c|\neg o)$. We can calculate this by conditioning on heartDisease, and then using the independence of chestPain from overweight given heartDisease.

$$\begin{aligned}
 P(c|\neg o) &= P(c|\neg o, h)P(h) + P(c|\neg o, \neg h)P(\neg h) \\
 &= P(c|h)P(h) + P(c|\neg h)P(\neg h) \\
 &= (0.3)(0.252) + (0.1)(1 - 0.252) \\
 &= 0.1504
 \end{aligned}$$



$$P(\text{smokes}) = 0.2$$

$$P(\text{overweight}) = 0.6$$

$$P(\text{heartDisease}) = \begin{cases} & \text{smokes} & \neg\text{smokes} \\ \text{overweight} & 0.5 & 0.3 \\ \neg\text{overweight} & 0.2 & 0.1 \end{cases}$$

	heartDisease	\neg heartDisease
$P(\text{chestPain})$	0.3	0.1

	heartDisease	\neg heartDisease
$P(\text{tiredness})$	0.5	0.4

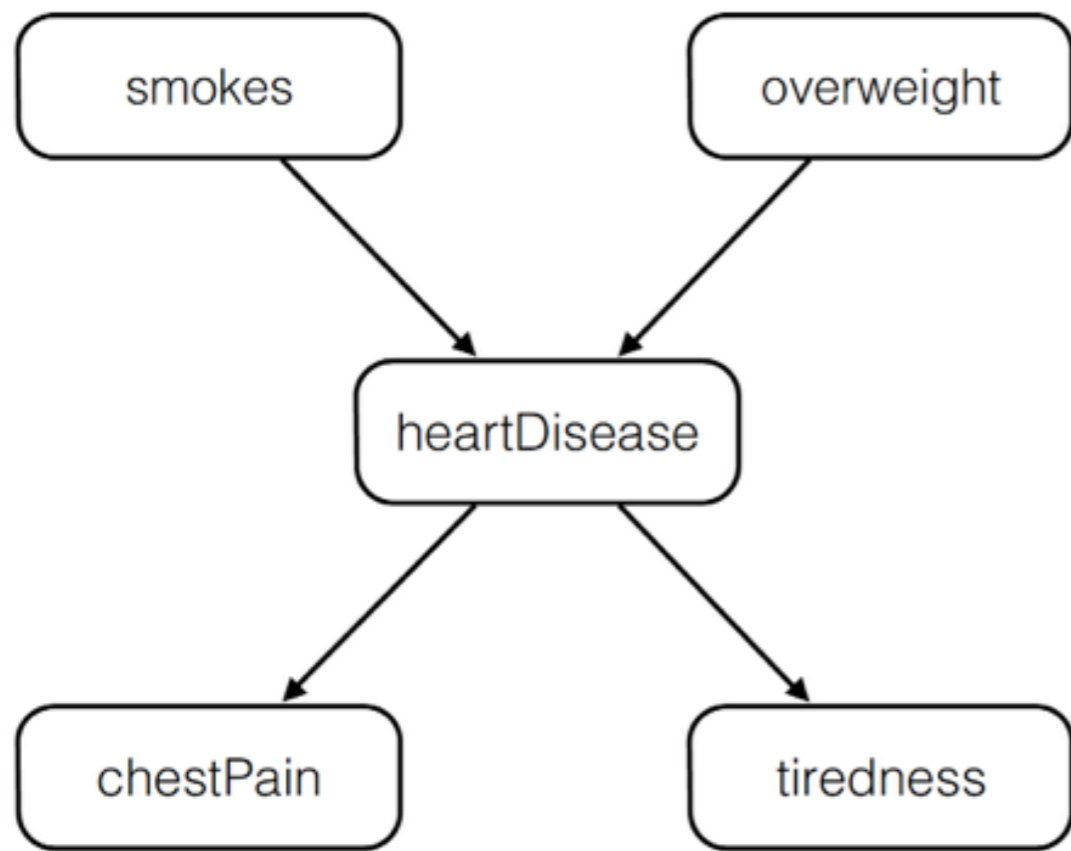
Calculate $P(\text{heartDisease} \mid \text{chestPain}, \neg\text{overweight})$

Solution: By Bayes law:

$$P(h|c, \neg o) = P(c|h, \neg o)P(h|\neg o)/P(c|\neg o)$$

Because chestPain is independent of overweight given heartDisease:

$$P(h|c, \neg o) = P(c|h)P(h|\neg o)/P(c|\neg o)$$



$$P(\text{smokes}) = 0.2$$

$$P(\text{overweight}) = 0.6$$

$$P(\text{heartDisease}) = \begin{cases} & \text{smokes} & \neg\text{smokes} \\ \text{overweight} & 0.5 & 0.3 \\ \neg\text{overweight} & 0.2 & 0.1 \end{cases}$$

	heartDisease	\neg heartDisease
$P(\text{chestPain})$	0.3	0.1

	heartDisease	\neg heartDisease
$P(\text{tiredness})$	0.5	0.4

Calculate $P(\text{heartDisease} \mid \text{chestPain}, \neg\text{overweight})$

$$\begin{aligned}
 P(h|c, \neg o) &= P(c|h)P(h|\neg o)/P(c|\neg o) \\
 &= (0.3)(0.12)/(0.1504) \\
 &= 0.2394
 \end{aligned}$$

Final Exam

- Neural Networks
 - Perceptron Training Rule
 - Sigmoid Unit Training Rule
- Decision Trees
 - Using Entropy & Information Gain
- A* Search

Final Exam

- Propositional Logic
 - Conversion to Clausal Form (CNF)
 - Resolution Refutation Proofs
- First Order Logic
 - Translating English to FOL
- Bayesian Networks

Final Exam

- May 9th, 4:00–6:00pm
- Bring a scientific calculator!
- Bring sharpened pencils!
- No notes, no computer, no cell phones!

