CSC242: Introduction to Artificial Intelligence
Homework 3: AIMA Chapter 7–9

1. Complete truth tables for the following formulas:
   (a) $\neg P$
   (b) $P \Rightarrow Q$
   (c) $\neg Q \Rightarrow \neg P$
   (d) $\neg P \Rightarrow \neg Q$
   (e) $P \wedge (Q \vee R)$
   (f) $(P \wedge Q) \vee (P \wedge R)$
2. Briefly and specifically define *entailment*. Why is it important?
3. Establish by model checking whether $P \Rightarrow Q \models \neg Q \Rightarrow \neg P$.
4. Establish by model checking whether $\{P, P \Rightarrow Q\} \models Q$.
5. Briefly define the following properties of a sentence or set of sentences:
   (a) Satisfiable
   (b) Unsatisatisfiable
   (c) Tautology
6. Briefly define the following properties of inference rules:
   (a) Soundness
   (b) Completeness
7. One rule of thumb for faculty hiring might be that a person who is not sociable ($\neg S$) is tenurable (T) if he or she is brilliant (B), but otherwise is not tenurable. Which of the following are correct representations of this assertion?
   (a) $(\neg S \wedge T) \iff B$
   (b) $\neg S \Rightarrow (T \iff B)$
   (c) $\neg S \Rightarrow ((B \Rightarrow T) \vee \neg T)$
8. Use resolution to prove the sentence $\neg A \wedge \neg B$ from the following set of clauses:

S1: $A \iff (B \vee E)$

S2: $E \Rightarrow D$

S3: $C \wedge F \Rightarrow \neg B$

S4: $E \Rightarrow B$

S5: $B \Rightarrow F$

S6: $B \Rightarrow C$

Hints: (1) Resolution requires conversion to a particular form. (2) To prove a conjunction it suffices to prove each conjunct separately.

9. Briefly explain why a knowledge base that can be expressed entirely as Horn clauses might be A Good Thing.

10. Briefly define the following terms related to first-order logic:
    (a) Domain or domain of discourse
    (b) Term
    (c) Atomic sentence or atomic formula

11. Describe the components of a first-order interpretation.

12. (a) Translate the following sentence of first-order logic into *good, natural* English:

$$\forall x, y, l \;\; \mathit{SpeaksLanguage}(x, l) \wedge \mathit{SpeaksLanguage}(y, l) \Rightarrow \mathit{Understands}(x, y) \wedge \mathit{Understands}(y, x). \tag{1}$$

    (b) Explain why this sentence is entailed by the sentence

$$\forall x, y, l \; \mathit{SpeaksLanguage}(x, l) \wedge \mathit{SpeaksLanguage}(y, l) \Rightarrow \mathit{Understands}(x, y). \tag{2}$$

    (c) Translate the following into first-order logic using the predicates *Understands* and *FriendOf*:
        i. Mutual understanding leads to mutual friendship.
        ii. Friendship is transitive (that is, my friend's friends are my friends also).

13. Write out the axioms for reasoning about the wumpus' location, using a constant symbol *Wumpus*, unary predicate *Smelly*, and binary predicates *In* and *Adjacent*. Hint: There is only one wumpus.

14. For each pair of atomic sentences, give the most general unifier if one exists:
    (a) $P(A, B, B)$ and $P(x, y, z)$
    (b) $Q(y, g(A, B))$ and $Q(g(x, x), y)$
    (c) $\mathit{Older}(\mathit{Father}(y), y)$ and $\mathit{Older}(\mathit{Father}(x), \mathit{John})$

(d) $Knows(Father(y), y)$ and $Knows(x, y)$

15. From "Horses are animals," it follows that "The head of a horse is the head of an animal." Demonstrate that this inference is valid by doing the following:
    (a) Translate the premise and the conclusion into first-order logic using the predicates $HeadOf(h, x)$ ("h is the head of x"), $Horse(x)$ ("x is a horse"), and $Animal(x)$ ("x is an animal").
    (b) Negate the conclusion, and convert the premise and the conclusion into conjunctive normal form.
    (c) Use resolution to show that the conclusion follows from the premise.

16. Suppose a knowledge base contains just the following first-order Horn clauses:

$$Ancestor(Mother(x), x)$$
$$Ancestor(x, y) \wedge Ancestor(y, z) \Rightarrow Ancestor(x, z)$$

    Consider a forward-chaining algorithm that, on the jth iteration, terminates if the KB contains a sentence that unifies with the query, and otherwise adds to the KB every atomic sentence that can be inferred from the sentences already in the KB after iteration $j - 1$.
    (a) For each of the following queries, say whether the algorithm will (1) give an answer (if so, give that answer); or (2) terminate with no answer; or (3) not terminate.
        i. $Ancestor(Mother(y), John)$
        ii. $Ancestor(Mother(Mother(y)), John)$
        iii. $Ancestor(Mother(Mother(Mother(y))), y)$
        iv. $Ancestor(Mother(John), Mother(Mother(John)))$
    (b) Can a resolution algorithm prove the sentence $\neg Ancestor(John, John)$?

17. Translate the following statement in FOL:

    - If Francis loves everything, then Francis is a saint.

    and then convert into CNF. Why does it make sense that the CNF version contains a Skolem function?

18. Translate the following statements into FOL, convert each to clausal form, and write a resolution refutation proof that answers the question.

    1. Jack owns a roomba.
    2. Every roomba owner is a robot enthusiast.
    3. No robot enthusiast breaks a robot.
    4. Either Jack or Jill broke my Roomba.

    - Question: Did Jill break my roomba?